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THE APPLICATION OF RELIABILITY-BASED DESIGN FACTORS IN STRESS CORROSION CRACKING EVALUATIONS

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ABSTRACT

First-order reliability methodology (FORM) is used to develop reliability-based design factors for deterministic analyses of stress corrosion cracking. The basic elements of FORM as applied to structural reliability problems are reviewed and then employed specifically to stress corrosion cracking evaluations. Failure due to stress corrosion cracking is defined as crack initiation followed by crack growth to a critical depth. The stress corrosion cracking process is thus represented in terms of a crack initiation time model and a crack growth rate model, with the crack growth rate integrated from the initiation time to the time at which the crack grows to its critical depth. Both models are described by log-normal statistical distribution functions. A procedure is developed to evaluate design factors that are applied to the mean values of the crack initiation time and the crack growth rate for specified temperature and stress conditions. The design factors, which depend on the standard deviations of the statistical distributions, are related to a target reliability, which is inversely related to an acceptable probability of failure. The design factors are not fixed, but are evaluated on a case-to-case basis for each application. The use of these design factors in a deterministic analysis assures that the target reliability will be attained and the corresponding acceptable probability of failure will not be exceeded. An example problem illustrates use of this procedure.

INTRODUCTION

Deterministic design-basis structural evaluations require the use of data that are often subject to considerable uncertainty. In some cases, the use of extreme bounding values for the relevant input parameters can lead to overly conservative results without any knowledge of the risk or, conversely, the reliability inherent in the use of those bounds. An approach that is gaining worldwide acceptance utilizes structural reliability methods to account for these uncertainties. Such methods allow for the calculation of design factors applied to the individual input variables that accommodate the uncertainties of the variables, which are characterized by statistical distribution functions, and an acceptable level of risk. Such methodology has been endorsed for the evaluation of flaws by the British Standards Institution in BS 7910 (1999) and the American Petroleum Institute in API 579 (2000). An ASME Boiler and Pressure Vessel Code Nuclear Code Case that deals with flaw evaluation also uses this methodology, which is commonly referred to as the "partial safety factor" treatment. The basis for the Code Case is discussed by Bloom (2000).

Typical stress corrosion cracking (SCC) design analysis procedures are based on statistical models for the crack initiation time and the crack growth rate. Both models are expressed in terms of log-normal distribution functions. In deterministic analyses, the design-basis SCC initiation time and crack growth rate values are traditionally established as fixed statistical bounds. For example, the design-basis SCC crack initiation time can be specified as the one-sided 90 percent lower bound of the initiation time model, while the design-basis crack growth rate, can be identified as the one-sided 95 percent upper bound of the SCC growth rate model. This typical set of bounds is referred to as "90/95 bounds".

Design-basis input parameters, such as those obtained from the fixed 90/95 bounds, can be expressed in terms of design factors applied to the mean values of the parameters (i.e., the 50 percent bounds). The specification of such design parameters presumes a small, but finite, risk of failure. Increasing the design factors is aimed at decreasing the probability of failure. A set of fixed SCC design factors, therefore, implies an acceptable probability of failure. Although this failure probability is not considered explicitly in a traditional deterministic analysis, its existence should be recognized. Furthermore, the acceptable failure probability implied by a set of fixed design factors is not fixed but depends on the details of the problem being evaluated. Implementation of reliability-based design principles, on the other hand, assures that an acceptable level of risk is not exceeded. This is accomplished by relating the design factors directly to a target structural reliability and the statistics of the input variables.

STRUCTURAL RELIABILITY METHODS

Haldar and Mahadevan (1995) give a rather complete summary of structural first-order and second-order reliability methods. The basis of such methods is the fundamental structural design criterion that requires a structure to have strength sufficient to resist its loading. Stated in its simplest terms, this means that the resistance, R, should exceed the loading, S. Defining a function $z = R - S$, $z < 0$ indicates failure while $z > 0$ designates no failure. The function z is commonly referred to as a "performance function" since the greater its value, the better the performance of the structure. Suppose that several of the variables that contribute to the performance function through the resistance, the loading, or both are associated with uncertainties and thus need to be treated as random variables, while the other variables are assumed to be fixed, known quantities not subject to uncertainty. Consider a structural system defined by n such random variables x_i, $i=1,2,3,\ldots,n$. Assume these variables to be both statistically independent and normally distributed. The performance function z, can then be expressed as a function of the x_i. Consider further the corresponding standard normal variables $x_i' = (x_i - \mu_i)/\sigma_i$, where μ_i and σ_i are the mean and standard deviation, respectively, of the x_i variable, so that

$$z = g(x_1', x_2', x_3', \ldots, x_n') \tag{1}$$

For a structure under prescribed loading, a surface defined by $z = 0$ can be constructed in n-dimensional space with the x_i' as the coordinates. The surface $z = 0$ is termed the limit surface. Values of the x_i' coordinates that result in a negative value of z indicate failure, while $z > 0$ characterizes no failure. This is illustrated in Fig. 1 in two-dimensional space (n=2). The limit curve $z = 0$ in Fig. 1 represents the locus of all combinations of the random variables x_1' and x_2' that result in imminent failure.

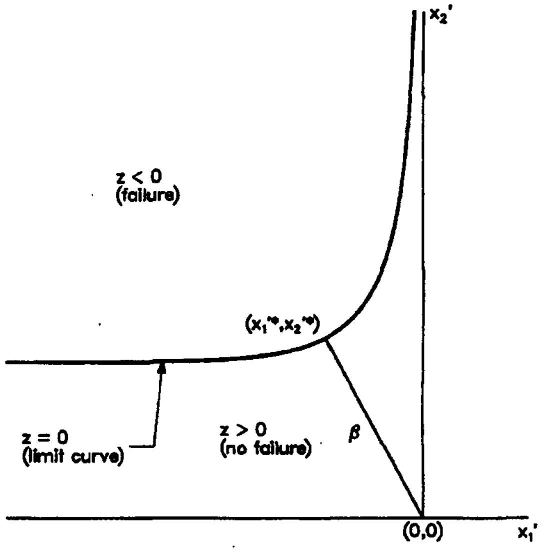

Figure 1 - Schematic of Limit Curve and Reliability Index

As discussed by Haldar and Mahadevan (1995), the performance function z in multi-dimensional space is expressed as a normally distributed function of the x_i' variables, with first-order approximations of the mean and variance as follows:

$$\mu_z = g(0,0,0,\ldots,0) \tag{2}$$

$$\sigma_z^2 = \sum_{i=1}^{n}\left[\frac{\partial g}{\partial x_i'}(0,0,0,\ldots,0)\right]^2 \tag{3}$$

The cumulative distribution function for the normal variable z is given by:

$$F(z) = \Phi\left(\frac{z-\mu_z}{\sigma_z}\right) \tag{4}$$

If the limit surface is linear, the probability of failure p_f, is represented by F(0), which is the area under the distribution curve for which $z < 0$. Therefore,

$$p_f = \Phi\left(-\frac{\mu_z}{\sigma_z}\right) \tag{5}$$

Letting $\beta = \mu_z/\sigma_z$ be the reliability index, the probability of failure can be expressed simply as:

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{6}$$

If the limit surface is nonlinear, as is generally the case, first-order reliability methods (FORM) are used to obtain a linear approximation to the limit surface at the point on the surface that is closest to the origin. Using FORM, β is the distance from the origin to this point whose coordinates are denoted by $x_i'^*$, i =1, 2, ..., n. Figure 1 illustrates this for two-dimensional space in which the limit surface is a curve. An iterative calculational algorithm is employed to determine β and an estimate of p_f is obtained by using this value of β in Eq. (6). The coordinates $x_i'^*$ are related to β by $x_i'^* = -\alpha_i^* \beta$, where α_i^* are the direction cosines.

Figure 1 shows that the smaller the reliability index β, the closer is the limit curve to the origin and the larger is the failure region. Thus, the position of the limit surface relative to the origin as characterized by β is a measure of the reliability of the system. Note that the origin represents the mean values of all of the random variables; i.e., $x_i' = 0$ (or $x_i = \mu_i$).

All points that constitute a limit surface correspond to the same level of reliability. The point on the surface closest to the origin (represented by the $x_i'^*$), however, represents the combination of variables $x_i^* = \mu_i + x_i'^* \sigma_i$, in closest proximity to the mean values. It is thus the optimum point for design since it represents the most probable failure point. Defining the design factor γ_i, for the variable x_i as the ratio of the optimum design variable to the mean value of the variable, $\gamma_i = x_i^*/\mu_i$,

$$\gamma_i = 1 + x_i'^* \sigma_i/\mu_i = 1 - \alpha_i^* \beta \, COV_i \qquad (7)$$

where $COV_i = \sigma_i/\mu_i$ is the coefficient of variation of the variable x_i.

STRUCTURAL RELIABILITY METHODS APPLIED TO STRESS CORROSION CRACKING

Definition of Failure

The first step in applying structural reliability methods is to define failure. Stress corrosion cracking failure is defined as crack growth to a specified critical depth. (This is a departure from the standard definition of the loading equal to the resistance.) Therefore, failure occurs when the time t_{CRIT}, required to grow a crack to its critical depth is less than the duration of the evaluation period, designated as t_{EOP}. Hence, the performance function z, is:

$$z = t_{CRIT} - t_{EOP} \qquad (8)$$

t_{CRIT} is the sum of two time periods: (1) the time to crack initiation, designated as t_i, and (2) the time period t_g, from crack initiation to growth of the crack to the critical depth. Therefore,

$$t_{CRIT} = t_i + t_g \qquad (9)$$

The SCC model presumes that the initiation time t_i, and the crack growth rate da/dt, are uncorrelated random variables described by log-normal distribution functions; all other input variables to an SCC evaluation are assumed to be known, fixed quantities.

Crack Initiation Time Model and Design Factor

The SCC initiation time model is represented as a log-normal distribution function in the form:

$$t_i = A_i \, (\sigma_i/\sigma_o)^{\tilde{n}} \exp(Q_i/RT) \exp[N(0,SD_i)] \qquad (10)$$

where

t_i = crack initiation time, days
A_i = mean crack initiation time coefficient, days
σ_i = surface stress at crack initiation site, ksi
σ_o = reference stress, ksi
$\tilde{n}$ = stress exponent
Q_i = crack initiation activation energy, kcal/mol
R = gas constant, 0.001987 kcal/mol·K
T = exposure temperature, K
N = normally distributed correlation error with mean = 0 and standard deviation = SD_i

The log-normal distribution for t_i can be expressed in terms of a standard normal distribution as follows:

$$t_i = t_{io} \exp(SD_i \, x_1') \qquad (11)$$

$t_{io} = A_i \, (\sigma_i/\sigma_o)^{\tilde{n}} \exp(Q_i/RT)$ is the mean value of the initiation time for the specified stress and temperature levels and x_1' is the standard normal variable associated with the initiation time model. The natural log of the initiation time coefficient is considered to be normally distributed so that the value of the coefficient A_i^{des}, that is used for design calculations is given by:

$$A_i^{des} = A_i \exp(SD_i \, x_1') \qquad (12)$$

Suppose for example that a fixed one-sided 90 percent lower bound were applied to the SCC initiation time model to establish a design-basis initiation time. From a normal distribution table, the 10th percentile of the normal distribution function corresponds to the standard normal variable $x_1' = -1.282$. The design-basis crack initiation time coefficient corresponding to this bound, therefore, is related to the mean value of the coefficient A_i, by:

$$A_i^{des} = A_i \exp(-1.282 \, SD_i) \qquad (13)$$

The quantity "$\exp(-1.282 \, SD_i)$" represents the design factor applied to the crack initiation time coefficient for the fixed 90 percent bound. In reliability-based design, however, x_1' is not fixed. From Eq. (12), the general expression for the design factor DF_i, on crack initiation time is given by:

$$DF_i = A_i^{des}/A_i = \exp(SD_i \, x_1') \qquad (14)$$

The design factor DF_i, is always less than one.

Crack Growth Rate Model and Design Factor

The SCC growth rate model is a log-normal distribution function given by:

$$da/dt = A_g\,(K_I/K_o)^n \exp(-Q_g/RT)\exp[N(0,SD_g)] \tag{15}$$

where

da/dt = crack growth rate, inches/day
A_g = mean crack growth rate coefficient, inches/day
K_I = applied stress intensity factor, ksi$\sqrt{\text{in}}$
K_o = reference stress intensity factor, ksi$\sqrt{\text{in}}$
n = stress intensity factor exponent
Q_g = crack growth rate activation energy, kcal/mol
R = gas constant, 0.001987 kcal/mol·K
T = exposure temperature, K
N = normally distributed correlation error with mean = 0 and standard deviation = SD_g

The time period t_g, from crack initiation to growth of the crack to its critical depth, is determined by integrating Eq. (15) from the initiation time t_i to the critical crack depth time t_{CRIT}. t_i is defined somewhat arbitrarily as the time when a crack has incubated and grown to an average depth of 0.010 inch, while t_{CRIT} is the time when the crack grows to its critical depth a_{CRIT}. The critical crack depth is determined by the criteria established to define failure.

The integration of Eq. (15) results in an expression for t_g in the form:

$$t_g = t_{go}\exp(-SD_g\,x_2') \tag{16}$$

t_{go} is the mean value of the crack growth period and x_2' is the standard normal variable associated with the SCC growth rate model. The natural log of the crack growth rate coefficient is considered to be normally distributed so that the value of the coefficient A_g^{des}, that is used for design calculations is given by:

$$A_g^{des} = A_g\exp(SD_g\,x_2') \tag{17}$$

Consider, as an example, a fixed one-sided 95 percent upper bound applied to the SCC growth rate model. The 95th percentile corresponds to the standard normal variable $x_2' = +1.645$. The design-basis crack growth rate coefficient corresponding to this bound, therefore, is related to the mean value of the coefficient A_g, by:

$$A_g^{des} = A_g\exp(+1.645\,SD_g) \tag{18}$$

The quantity "$\exp(+1.645\,SD_g)$" represents the design factor applied to the crack growth rate coefficient for the 95 percent bound. The variable x_2', however, is not fixed when using reliability-based design. From Eq. (17), the general expression for the design factor DF_g, on crack growth rate is given by:

$$DF_g = A_g^{des}/A_g = \exp(SD_g\,x_2') \tag{19}$$

The design factor DF_g, is always greater than one.

Synthesis of Crack Initiation and Crack Growth Rate Models

The expressions for the crack initiation time t_i, and the subsequent period of crack growth to the critical depth t_g, given by Eqs. (11) and (16), respectively, are log-normal distributions that depend on the standard deviations for the respective models and the bounds selected for design calculations. The development of design factors based on structural reliability methodology requires the synthesis of these distributions. Although some of the other input parameters necessary to determine t_i and t_g may be associated with uncertainties, they are assumed to be known, fixed quantities.

Eqs. (11) and (16) are substituted into Eq. (9) to give the following equation for the total time required to grow a crack to its critical depth:

$$t_{CRIT} = t_{io}\exp(SD_i\,x_1') + t_{go}\exp(-SD_g\,x_2') \tag{20}$$

Note that t_{CRIT} evaluated for design calculations depends on the bounds chosen for both the crack initiation time and crack growth rate models as well as the standard deviations associated with the models. The best-estimate value of t_{CRIT} is obtained by setting $x_1' = x_2' = 0$, while the design factors are implemented by specifying the values of x_1' ($x_1' < 0$) and x_2' ($x_2' > 0$).

Relationship of Specified Design Factors to Structural Reliability and Probability of Failure

We now wish to determine the probability of failure when, for a specified set of design factors characterized in terms of x_1' and x_2', the critical crack depth time t_{CRIT}, is equal to the exposure time t_{EOP}. This is the failure probability if failure calculated deterministically using these design factors were to occur precisely at the end of the evaluation period. Since t_{CRIT} depends on the set of bounds chosen for the crack initiation time and the crack growth rate (i.e., on x_1' and x_2'), the failure probability is tied to the selected design factors.

Substituting Eq. (20) into Eq. (8) for the performance function z:

$$z = g(x_1', x_2') = t_{io}\exp(SD_i\,x_1') + t_{go}\exp(-SD_g\,x_2') - t_{EOP} \tag{21}$$

z is expressed as a function $g(x_1', x_2')$ of the two standard normal variables x_1' and x_2'. This shows that a structure under specified temperature and stress conditions can be subject to imminent failure at a specified exposure time t_{EOP}, for any number of combinations of the crack initiation time and crack growth rate coefficients that result in $z = 0$. These combinations all produce the same failure time, t_{CRIT}. This is illustrated by the limit curve shown in Fig. 1.

From Eq. (21), the limit curve defining imminent failure depends on the best-estimate crack initiation time, the best-estimate time period from crack initiation to growth to the critical crack depth, the standard deviations of the crack initiation time and crack growth rate models, and the duration of the evaluation period (exposure time). The limit curve not only defines the combinations of crack initiation time and crack growth rate coefficients that result in failure at a specified exposure time, but also provides a measure of the reliability of the system under the specified conditions of temperature and stress. The reliability is measured by the proximity of the limit curve to the origin $(x_1', x_2') = (0, 0)$. The closer the limit curve to the origin, the larger the failure region ($z < 0$) and, therefore, the probability of failure. (A limit curve passing through the origin would be a best-estimate limit curve associated with a failure probability of 0.5.) The reliability of the system is characterized by the shortest distance β, from the origin (0, 0) to the limit curve at point $(x_1'^*, x_2'^*)$.

For a structure subject to specified temperature and stress conditions, a set of design coefficients will result in failure at some exposure time t_{EOP}, that results in $z = 0$. Thus, the specified design conditions are associated with a specific level of reliability; i.e., the reliability index β, and, from Eq. (6), a corresponding failure probability p_f. This ties the selected design factors to the reliability of the component for the specified conditions.

Conversely, for a specified reliability β, the most likely or most probable set of design coefficients is the one that is in closest proximity to the set of best-estimate coefficients. The optimum set corresponds to the point $(x_1'^*, x_2'^*)$ on the limit curve that is closest to the origin (0, 0), which from Eq. (20) represents the best-estimate, or most likely, set of conditions for failure. Note, however, that this set is unique and needs to be determined on an individual basis for a structure subject to specified temperature and stress conditions since the limit curve depends on these conditions.

RELIABILITY-BASED CRACK INITIATION TIME AND CRACK GROWTH RATE DESIGN COEFFICIENTS

The development of reliability-based crack initiation time and crack growth rate design coefficients is predicated on the specification of a target reliability index β_T that is associated with an acceptable level of risk expressed in terms of a probability of failure in accordance with Eq. (6). Once the target reliability is established, the design factors and design coefficients are developed.

Evolution of Design Factors and Design Coefficients

The limit curve can be expressed in a dimensionless form as $z' = 0$, where $z' = z/t_{go}$, so that:

$$z' = t_{io}/t_{go} \exp(SD_i\, x_1') + \exp(-SD_g\, x_2') - t_{EOP}/t_{go} = 0 \qquad (22)$$

The optimum crack initiation time and crack growth rate design factors and the resulting design coefficients are calculated by observing from Eq. (22) and Fig. 1 that, for the target reliability index β_T, the limit curve is uniquely determined from the following three parameters:

- the standard deviation SD_i, of the log-normal crack initiation time model;
- the standard deviation SD_g, of the log-normal crack growth rate model; and
- the ratio t_{io}/t_{go}, of the best-estimate time to crack initiation to the best-estimate crack growth time from initiation to failure.

The term t_{EOP}/t_{go} assumes whatever value is required to achieve a distance $\beta = \beta_T$ from the origin to the limit curve. Therefore, the optimum design factors from the reliability-based procedure depend on the particular application. The ratio t_{io}/t_{go} is determined on an individual basis for an SCC problem by calculating the best-estimate times.

The following procedure establishes a general set of design factor curves:

1. For a specified target reliability β_T, construct limit curves for a wide range of ratios t_{io}/t_{go}. To cover all possible contingencies, curves have been developed for the range $0.001 \le t_{io}/t_{go} \le 1000$. A very low ratio signifies relatively early initiation ($t_{io}/t_{go} = 0$ corresponds to an assumption of initiation at time zero, which is the case for a known, pre-existing crack), while a very high ratio indicates that the crack initiaton time comprises the bulk of the time to failure. The limit curves are constructed using an iterative procedure with the ratio t_{EOP}/t_{go} varied until $\beta = \beta_T$ (see Fig. 1).
2. For each value of the ratio t_{io}/t_{go}, determine the most likely, or optimum, set of standard normal variables. This set is defined by the point $(x_1'^*, x_2'^*)$ on the limit curve that is closest to the origin (0, 0). This point is at a distance β_T from the origin. Generate curves of $x_1'^*$ vs. t_{io}/t_{go} and $x_2'^*$ vs. t_{io}/t_{go}.
3. Determine the bounds associated with the optimum normal variables. The optimum normal variable $x_1'^*$ governs the optimum lower bound on the crack initiation time model, while the variable $x_2'^*$ controls the optimum upper bound on the crack growth rate model. The bounds are determined by the probabilities $p(x_1'^*)$ and $p(x_2'^*)$ from a normal probability table. For example, the set $(x_1'^*, x_2'^*) = (-1.282, +1.645)$ results in $p(x_1'^*) = 0.10$ and $p(x_2'^*) = 0.95$. The former represents the one-sided 90 percent lower bound on the crack initiation time model, while the latter corresponds to the one-sided 95 percent upper bound on the crack growth rate model. Generate curves of $p(x_1'^*)$ vs. t_{io}/t_{go} and $p(x_2'^*)$ vs. t_{io}/t_{go}.

4. Evaluate the optimum design factors for crack initiation time and crack growth rate from Eqs. (14) and (19), respectively. That is,

$$DF_i = \exp(SD_i\, x_1'^*) \qquad (23)$$

$$DF_g = \exp(SD_g\, x_2'^*) \qquad (24)$$

Generate curves of DF_i vs. t_{io}/t_{go} and DF_g vs. t_{io}/t_{go}.

The most likely, or optimum, crack initiation time and crack growth rate design coefficients are determined for a specified value of t_{io}/t_{go} by:

$$A_i^{des} = A_i\, DF_i \qquad (25)$$

$$A_g^{des} = A_g\, DF_g \qquad (26)$$

Example of Reliability-Based Bounds and Design Factors

Figure 2 displays plots of the optimum standard variables $x_1'^*$ and $x_2'^*$ as functions of the best-estimate ratio of crack initiation time to crack growth time t_{io}/t_{go}, for a target reliability $\beta_T = 1.875$ and standard deviations $SD_i = 2.096$ and $SD_g = 0.701$. From Eq. (6), a target reliability $\beta_T = 1.875$ corresponds to an acceptable level of risk $p_f = 0.03$. Figure 3 gives plots of the corresponding optimum bounds on the crack initiation time and crack growth rate models. Figures 4 and 5 show plots of the optimum design factors DF_i and DF_g, respectively, for $\beta_T = 1.875$.

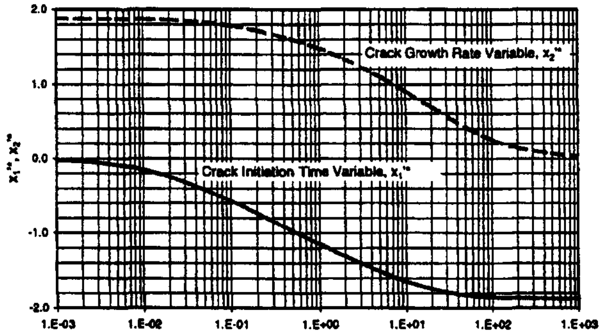

Figure 2 – Optimum Crack Initiation Time and Crack Growth Rate Standard Normal Variables

Figure 2 shows that the optimum standard normal variables $x_1'^*$ and $x_2'^*$, associated with the crack initiation time model and the crack growth rate model, respectively, depend quite strongly on the best-estimate time to crack initiation relative to the crack growth time from initiation to failure. The set of optimum normal variables for $\beta_T = 1.875$ varies between the limits $(x_1'^*, x_2'^*) = (0.0, 1.875)$ when $t_{io}/t_{go} \to 0$ and $(x_1'^*, x_2'^*) = (-1.875, 0.0)$ as $t_{io}/t_{go} \to \infty$. From Fig. 3, the corresponding limiting values of the optimum bounds are $p(x_1'^*) = 0.5$, $p(x_2'^*) = 0.970$ when $t_{io}/t_{go} \to 0$, and $p(x_1'^*) = 0.030$, $p(x_2'^*) = 0.5$ as $t_{io}/t_{go} \to \infty$.

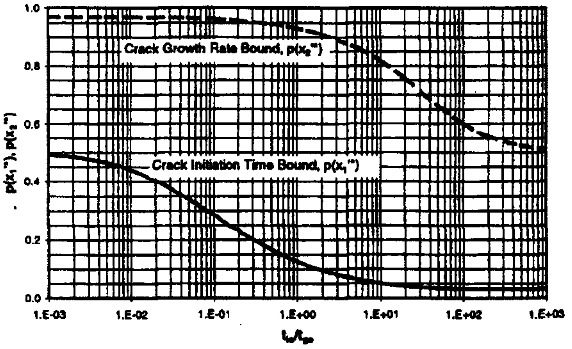

Figure 3 – Optimum Crack Initiation Time and Crack Growth Rate Bounds

The $t_{io}/t_{go} \to 0$ limits show that if the crack initiation time is very small and the time to failure consists almost entirely of growth of the crack to its critical depth, the optimum bound on the initiation time model approaches the 50 percent bound (i.e., the best-estimate initiation time is optimum), while the optimum upper bound on the crack growth rate model approaches 97 percent.

The $t_{io}/t_{go} \to \infty$ limits, on the other hand, indicate that if the time to failure consists almost entirely of the time to crack initiation with growth of the crack to its critical depth occurring very quickly, the optimum lower bound on the crack initiation time model approaches 97 percent $[p(x_1'^*) = 0.030]$ for $\beta_T = 1.875$. The optimum bound on the crack growth rate model approaches the 50 percent best-estimate bound.

If the best-estimate total time to failure were evenly split between the time to crack initiation and the growth time from initiation to failure (i.e., $t_{io}/t_{go} = 1$), the optimum initiation time/growth rate bounds are 87.4/93.1. Use of these bounds under these conditions ensure that the target reliability $\beta_T = 1.875$ is maintained and the corresponding acceptable level of risk is not exceeded.

The optimum bounds on the crack initiation time and crack growth rate models are not fixed quantities, but are functions of the best-estimate ratio of the time to crack initiation to the time period from initiation to growth of the crack to its critical depth. This process assures that the target reliability is attained in all cases, and that the most likely design-basis initiation and failure times corresponding to the target reliability are calculated.

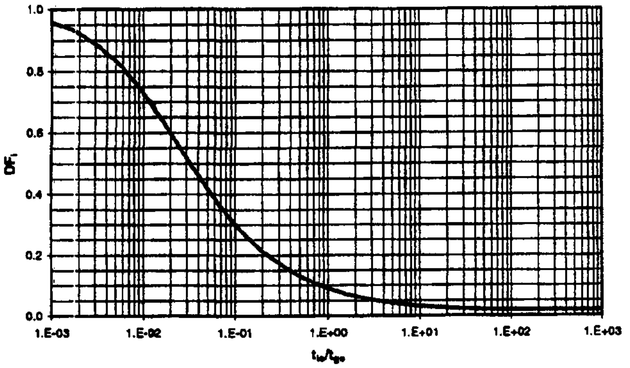

Figure 4 – Optimum Crack Initiation Time Design Factor

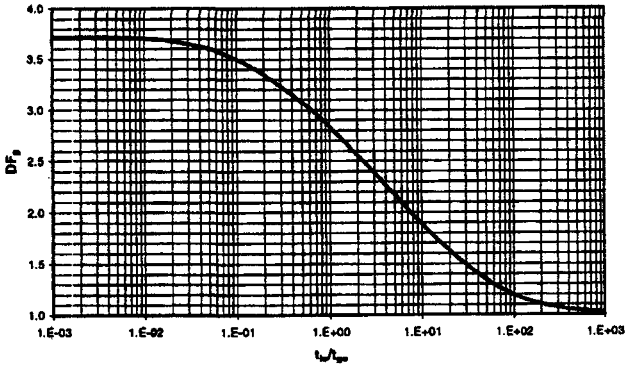

Figure 5 – Optimum Crack Growth Rate Design Factor

Figure 4 shows that the design factor DF_i, applied to the best-estimate crack initiation time coefficient varies from 1.0 at $t_{io}/t_{go} = 0$ to 0.020 for large values of t_{io}/t_{go}. Hence, when the best-estimate time to crack initiation is very short relative to the total time to failure, the initiation time design factor approaches unity as a result of the decreased importance of crack initiation relative to crack growth. The limiting case would be the analysis of a pre-existing stress corrosion crack for which only a crack growth calculation would be conducted. At the other extreme, the design factor on the crack initiation time is quite significant if the total time to failure consists mostly of the time to initiation and crack growth to the critical crack depth occurs relatively quickly.

Figure 5 shows that the design factor DF_g, applied to the best-estimate crack growth rate coefficient varies from the asymptotic value 3.722 as $t_{io}/t_{go} \rightarrow 0$ to 1.0 for large values of t_{io}/t_{go}. When the bulk of the total time to failure consists of the time to initiation and crack growth to the critical crack depth occurs quickly, the crack growth rate design factor approaches unity as a result of the decreased importance of crack growth relative to crack initiation. At the other extreme, the design factor on crack growth rate is relatively large if the total time to failure consists mostly of crack growth.

ILLUSTRATIVE APPLICATIONS

The curves in Fig. 2 are used to determine the optimum set of standard normal variables ($x_1'^*$, $x_2'^*$) for the target reliability $\beta_T = 1.875$ and standard deviations $SD_I = 2.096$ and $SD_g = 0.701$. The corresponding optimum bounds on crack initiation time and crack growth rate, $p(x_1'^*)$ and $p(x_2'^*)$, respectively, are plotted in Fig. 3, while Figs. 4 and 5 show plots of the design factors DF_i and DF_g, respectively, applied to the best-estimate values of the crack initiation time and crack growth rate coefficients. The steps necessary to determine the reliability-based design factors are:

1. Perform a best-estimate calculation to determine the crack initiation time t_{io}, the crack growth time t_{go}, and the ratio t_{io}/t_{go}.

2. From the $x_1'^*$ vs. t_{io}/t_{go} and $x_2'^*$ vs. t_{io}/t_{go} curves plotted in Fig. 2, determine $x_1'^*$ and $x_2'^*$ for the best-estimate ratio t_{io}/t_{go} calculated in Step 1.

3. From either the $p(x_1'^*)$ vs. t_{io}/t_{go} and $p(x_2'^*)$ vs. t_{io}/t_{go} curves plotted in Fig. 3 or a normal probability table, determine $p(x_1'^*)$ and $p(x_2'^*)$ for the best-estimate ratio t_{io}/t_{go} calculated in Step 1. This is an optional step that establishes the statistical bounds and thus serves to provide some insight into the results.

4. Determine DF_i and DF_g for the best-estimate ratio t_{io}/t_{go} calculated in Step 1. DF_i is evaluated from either the design factor curve for crack initiation time DF_i vs. t_{io}/t_{go} plotted in Fig. 4 or Eq. (23), while DF_g is determined either from the Fig. 5 curve for crack growth rate DF_g vs. t_{io}/t_{go} or Eq. (24).

Table 1 gives the results of these calculations for two illustrative problems. Problem 1 illustrates the generation of reliability-based design factors for a case in which the total time to failure calculated on a best-estimate basis consists mostly of the time to initiation and crack growth to the critical crack depth occurs relatively quickly. Problem 2, on the other hand, applies to the case of the best-estimate total time to failure split rather evenly between the time to crack initiation and the growth time from initiation to failure.

Table 1. Design Factor Results for Illustrative Problems

Problem	1	2
t_{io} (days)	30,970	4,650
t_{go} (days)	5,390	4,960
t_{io}/t_{go}	5.746	0.938
$x_1'*$	-1.540	-1.128
$x_2'*$	1.069	1.498
$p(x_1'*)$	0.062	0.130
$p(x_2'*)$	0.858	0.933
DF_i	0.040	0.094
DF_g	2.116	2.858
t_i (days)	1,230	440
t_g (days)	2,550	1,570
t_i+t_g (days)	3,780	2,010

The information given in Table 1 leads to the following observations on the application of statistical bounds for the two problems:

Problem 1. The calculated best-estimate ratio $t_{io}/t_{go} = 5.746$. The best-estimate total time to failure is comprised mostly of the time to crack initiation; the crack growth time period from initiation to failure is relatively short. The importance of crack initiation relative to crack growth is enhanced in this case and the optimum lower bound applied to the initiation time model, therefore, is more stringent. The optimum lower bound on the initiation time model is 93.8 percent [$p(x_1'*) = 0.062$], while the optimum upper bound on the crack growth rate model is 85.8 percent.

The design factors applied to the best-estimate crack initiation time and crack growth rate depend strongly on the respective variances of these variables. The relatively high standard deviation of the crack initiation time distribution function produces a design-basis crack initiation time that for this case is a factor of 25 lower than the best-estimate crack initiation time. The design-basis crack growth time, however, is only a factor of 2.1 lower than the best-estimate crack growth time. This illustrates the influence of the statistics of the individual input variables on the calculated design factors.

Problem 2. The best-estimate crack initiation time is slightly less than the crack growth to failure time, so that the ratio $t_{io}/t_{go} = 0.938$. Hence, the importance of crack growth is on a par with that of crack initiation. The result is an optimum lower bound of 87.0 percent on the initiation time model and an optimum upper bound of 93.3 percent on the crack growth rate model.

As with Problem 1, the relatively high variance associated with the crack initiation time model yields a design-basis crack initiation time that is much less than the best-estimate time, although in this case the factor is 10.6. The design-basis crack growth time is a factor of 3.2 lower than the best-estimate crack growth time. The results of this problem illustrate the effects of both the input variable statistics and the best-estimate crack growth time relative to the crack initiation time.

General Observation. These problems demonstrate that the design-basis crack initiation time and crack growth rate design factors and coefficients obtained using reliability-based methodology are not fixed as they are in the traditional deterministic procedures. Rather, for a specified target reliability, they are influenced by the relative contributions to the time to failure of the initiation time and the time from crack initiation to growth to the critical depth, as well as by the statistics of the input variables.

CONCLUSIONS

1. First-order reliability methodology is a valuable tool for relating design factors used in deterministic structural analysis to the structural reliability expressed in terms of a reliability index. This approach utilizes the statistics of the key input parameters and aids in the performance and interpretation of the results of sensitivity studies. The reliability methods are used to determine an optimum set of design factors or statistical bounds corresponding to a specified reliability index. These bounds are the most probable values of the input parameters associated with the failure mechanism of the structure under specified loading and temperature conditions.

2. Methodology is in place to relate design factors used in deterministic evaluations of stress corrosion cracking to a specified level of risk. The level of risk is characterized by a probability of failure that is inversely related to the reliability index. The statistics of the input parameters to which design factors are assigned (in this case, the crack intiation time and the crack growth rate) enable the effects of each parameter on the reliability index or the risk to be determined. A set of design factors implies an acceptable level of risk and an acknowledgement that the probability of failure, although small, is finite.

3. Crack initiation time and crack growth rate design factors applied to the respective models are developed using reliability-based methods to assure that in all cases and under any conditions of temperature, stress, and geometry, a known, acceptable level of risk is assured. The statistical bounds used to generate the design factors, however, are not fixed. They are problem-dependent and determined specifically for each application.

4. A target reliability index related to an acceptable probability of failure must be specified in order to develop reliability-based design factors.

5. For a specified target reliability, design factors on crack initiation time and crack growth rate are determined from (1) the standard deviation of the log-normal crack initiation time model, (2) the standard deviation of the log-normal crack growth rate model, and (3) a best-estimate calculation of the

ratio of the crack initiation time to the time from initiation to growth of the crack to a critical depth that defines failure.

6. Implementation of the reliability-based procedure requires two sets of calculations: (1) best-estimate calculations to determine the ratio of crack initiation time to crack growth time and thus the design factors, and (2) design-basis calculations to determine crack initiation time and failure time using the computed design factors.

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